Roll No. .... Total Pages : 3

BT-4/J-22 **44151**

DISCRETE MATHEMATICS

Paper–PC-CS-202A

[Time : Three Hours] [Maximum Marks : 75]

Note : Attempt *five* questions in all, selecting at least *one* question from each unit.

UNIT–I

1. (a) Using mathematical induction, prove that n^3+2n is divisible by 3.

(b) Prove that $(A \cup B)' = A' \cap B'$

2. (a) Construct the truth tables for the following statements:

(i) $\neg(p \wedge q) \vee (\neg r)$.

(ii) $\neg(p \wedge \neg q) \vee (r)$.

(b) If set A is finite and contains n elements, prove that the power set of P(A) of the set A contains 2^n elements.

UNIT–II

3. (a) Consider the relation

R={(a,b) | length of string a= length of string b}

on set of strings of English letters. Prove that R is an equivalent relation.

(b) Show that the inclusion relation $\subseteq$ is a partial ordering relation on the power set of a set.

4. (a) Given A={1, 2, 3}, B={a,b} and C={l, m, n}. Find each of the following sets:

(i) A × B × C

(ii) A × C

(iii) B × C × A

(b) Define Lattice. Prove that D_{36} the set of divisors of 36 ordered by divisibility forms a lattice.

UNIT–III

5. (a) Prove that function $f : N \rightarrow N$ defined as

f(n) = {

n + 1, if n is odd

n - 1, if n is even

}

is inverse of itself.

(b) Solve: $a_n + a_{n-1} = 3n2^n$, $a_0 = 0$, using Generating function method.

6. (a) Let $f : Z \rightarrow Z$ be defined by $f[x] = 3x^3 - x$. Is this function:

(i) One-to-one?

(ii) Onto?

(b) There are 280 people in the party. Without knowing anybody's birthday, what is the largest value of n for which we can prove that at least n people must have been born in the same month?

UNIT–IV

7. (a) Prove that the identity element in a group is unique.

(b) Let G be the group and $a \in G$. Prove that the cyclic subgroup of $N(a) = \{x \in G : xa = ax\}$.

8. (a) Let P be a subgroup of a group G and let $Q = \{x \in G : xP = Px\}$.
Is Q a subgroup of G?

(b) Let $f : (R,+) \rightarrow (R_+, \times)$ is defined as $f(x) = e^x$ for all x in R, where $R \rightarrow$ set of real numbers and $R_+ \rightarrow$ set of positive real numbers. Prove that f is a homomorphism. Is f an isomorphism?